Modelling and identification of impulse responses for linear time invariant thermal systems stimulated by a unique separable heat source

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The Laplace transformation is commonly used for modeling dynamical systems. It allows transforming an ordinary differential equation (ODE) into a non differential one. If time is the independent variable, and if both the equation and its initial conditions are linear, with time invariant coefficients (LTI), the Laplace transform of the solution is proportional either to the transform of the source term, that is the variation of its intensity with time (case of a forced response) or to the non-zero initial condition(s) (relaxation case). If one gets back to the time domain, the forced solution is a convolution product between the original (inverse transform) of the transfer function, that is the impulse response, and the intensity of the source. This property can be extended to space discretized problems that are governed by a system of ODEs with initial conditions (electrical circuits in transient regime for example).

For partial differential equations (PDE) systems under LTI assumptions, such as conduction heat transfer, some analytical solutions, in the Laplace domain, can be derived for configurations where boundary conditions apply over simple geometrical surfaces (slabs, cylinders, spheres, ...), see the Thermal Quadrupole method for example [1]. They are now complemented by numerical inversion algorithms of the corresponding transforms at discrete times to recover the transient detailed temperature/flux fields.

The Laplace transformation can also be applied to physical systems where heat transfer is governed by a PDE system, but with non simple geometries: it is the case for a heterogeneous physical system (including solids and flowing fluid, even with linearized radiation in a cavity configuration) under LTI assumptions (time constant, but not necessary uniform, velocity field), in any 3D geometry. This also requires each single transient source (the input) to be separable, which means it can be written, in the transient heat equation and in its boundary conditions as a product of a time part, its intensity, by a space part, its geometrical support [2]. The temperature response at any point (output) is still a convolution product in time between its cause (source or input), and the impulse response.

In practice, the impulse response has to be found through solving an inverse problem, here a deconvolution. One can either use a numerical temperature solution of a Finite Element simulation code for a given source (model reduction), or the experimental noisy temperature signal delivered by a local sensor for a measured source in a calibration experiment (model identification, which requires some kind of regularization in the inversion). Examples of experimentally identified impulse responses, for characterizing heat exchangers [3], are given in this presentation: impedances, between a temperature and a thermal power, or transmittances, linking temperatures at two different points.

References


KEYWORDS: Laplace transform; convolution product; impedance; model reduction; model identification; heat transfer